WA Exams Practice Paper C, 2015

Question/Answer Booklet

MATHEMATICS METHODS UNITS 1 AND 2

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed **(98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(5 marks)**

In a random survey of 162 swimmers at a council owned pool, 109 said they swam regularly. 39 males said they swam regularly and 16 fewer males than females were surveyed.

(a) Complete this two-way table using the above information. (2 marks)

	Swam regularly	Did not swim regularly	Total
Female	70	19	89
Male	39	34	73
Total	109	53	162

(b) If one swimmer is selected at random from those surveyed, determine the probability

(i) they swam regularly. (1 mark)

$$\frac{109}{162} \approx 0.673$$

(ii) they swam regularly, given that they were female. (1 mark)

$$\frac{79}{89} \approx 0.787$$

(c) Based on the information in the table, is there any indication that swimming regularly at the council owned pool is independent of the gender of the swimmer? Justify your answer. (1 mark)

No, as from the probabilities calculated in (b) it can be seen that $P(SR) \neq P(SR \mid F)$.

Question 9 **(9 marks)**

(a) An arithmetic sequence has third term 28 and eighth term 41.75.

(i) Determine a definition of this sequence in the form $T_n = a + (n-1)d$. (2 marks)

$$d = \frac{41.75 - 28}{8 - 3} = 2.75$$
$$a = 28 - 2 \times 2.75 = 22.5$$
$$T_n = 22.5 + (n-1) \times 2.75$$

(ii) Determine the sum of the first twenty terms of this sequence. (1 mark)

$$S_{20} = \frac{20}{2}(2 \times 22.5 + (20-1) \times 2.75)$$
$$= 972.5$$

(b) A book editor charged clients 65 cents per page plus a flat fee of $120.

(i) Determine a recursive rule for the amount, a_n, the editor charges to edit a book of n pages, where a_n is in dollars. (2 marks)

$$a_n = a_{n-1} + 0.65$$
$$a_0 = 120$$

(ii) The editor charged a client less than $220 to edit a draft manuscript. Determine the maximum number of pages the draft contained. (1 mark)

$$120 + n \times 0.65 \le 220 \implies n \le 153$$

(c) Determine T_{10} of the arithmetic sequence where $T_1 = x - 3$, $T_2 = 2x + 1$ and $T_3 = 4x - 1$. (3 marks)

$$d = 4x - 1 - (2x + 1) = 2x + 1 - (x - 3) \implies x = 6$$

$$d = 10$$
$$a = 6 - 3 = 3$$

$$T_{10} = 3 + 9 \times 10$$
$$= 93$$

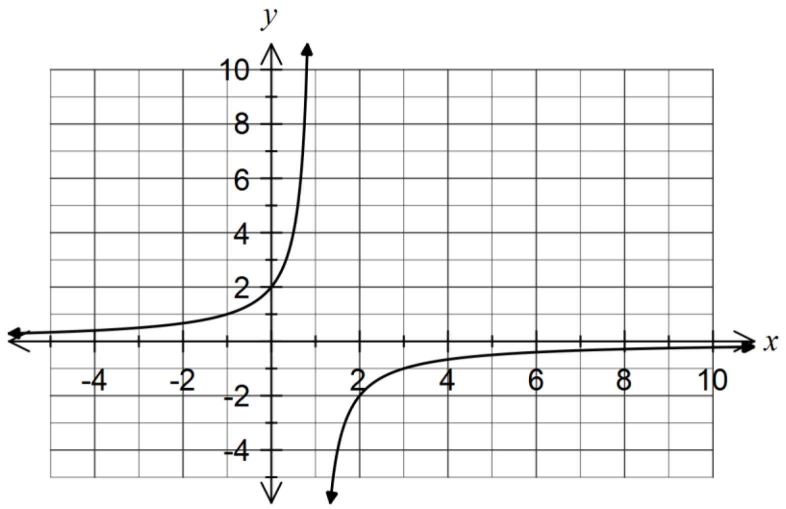
Question 10 **(8 marks)**

(a) Let $f(x)=\dfrac{2}{1-x}$.

(i) State the equations of the asymptotes of the graph of $y=f(x)$. (2 marks)

$x=1$ and $y=0$

(ii) Sketch the graph of $y=f(x)$. (2 marks)



(b) The two variables h and w are inversely proportional to one another.

(i) Circle all of the equations below that reflect this relationship, where k is a constant. (2 marks)

$h+w=k$ $w=hk$ $\boxed{wh=k}$ $\dfrac{h}{k}=w$ $\boxed{\dfrac{k}{w}=h}$ $\dfrac{h}{w}=k$

(ii) When $h=12.5$, $w=38.8$. If h decreases by 2.8, by how much will w change? (2 marks)

$12.5-2.8=9.7$

$12.5\times 38.8=9.7\times w$
$w=50$

$50-38.8=11.2$

w will increase by 11.2

Question 11 **(10 marks)**

(a) Determine x if the terms 12, x, 27 form part of a geometric sequence. (2 marks)

$$x^2 = 12 \times 27$$
$$x = \pm 18$$

(b) A team of workers is using a pile driver to drive wooden poles 4 metres long into the ground. The first hit of the pile driver drives a pole 50 cm into the ground. The second hit drives the pole another 40 cm into the ground. The third hit drives the pole another 32 cm into the ground and successive distances driven by the pile driver form a geometric sequence.

(i) How much further will the fourth hit drive the pole into the ground? (1 mark)

$$40 \div 50 = 0.8$$
$$32 \times 0.8 = 25.6 \text{ cm}$$

(ii) Determine the total distance the wooden pole has been driven into the ground after 12 hits of the pile driver. (2 marks)

$$S_{12} = \frac{50(1-0.8^{12})}{1-0.8} = 232.82 \text{ cm}$$

(iii) If the workers continued in this way for some time, what length of the wooden pole will always be left above the ground? Justify your answer. (2 marks)

$$S_{\infty} = \frac{50}{1-0.8} = 250$$

$$400 - 250 = 150 \text{ cm}$$

(c) A geometric series has a first term of 80 and a sum to infinity of 800. Determine the minimum number of terms of this sequence required so that their sum exceeds 700. (3 marks)

$$800 = 80 \div (1-r) \Rightarrow r = 0.9$$

$$\frac{50(1-0.9^{n})}{1-0.9} \geq 700 \Rightarrow n \geq 20$$

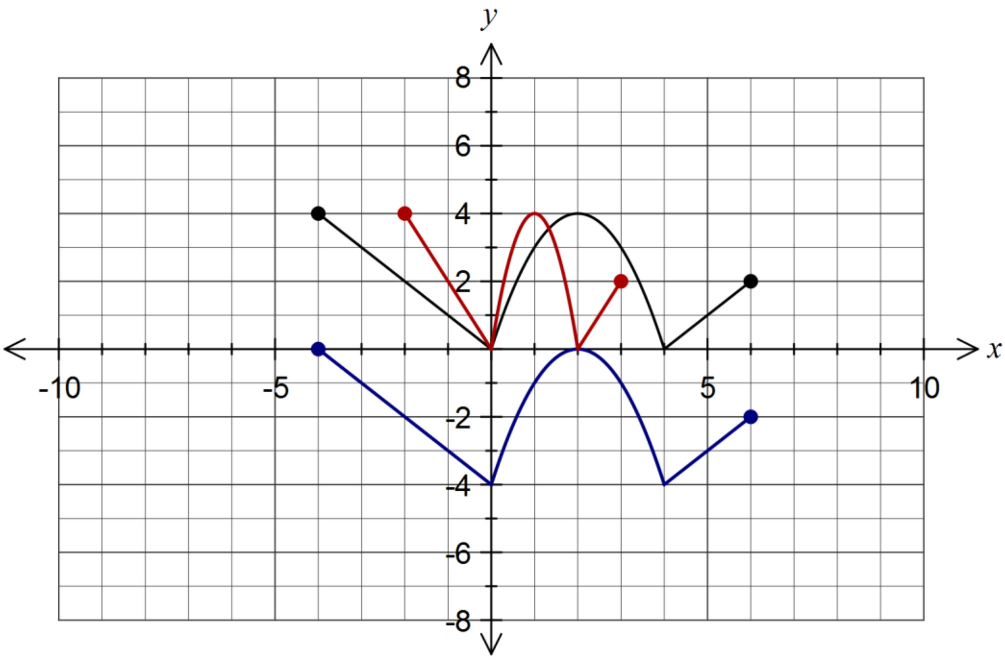
Question 12 **(5 marks)**

(a) State the domain and range of $f(x)=x^4-4$. (2 marks)

$x: x \in \mathbb{R}$

$y: y \geq -4$

(b) The graph of $y=g(x)$ is shown below.



On the same axes, sketch the graph of

(i) $y=g(x)-4$. (1 mark)

(ii) $y=g(2x)$. (2 marks)

Question 13 **(12 marks)**

(a) A thin piece of glass has been cut into the shape of an obtuse-angled triangle with an area of 135.5 cm^2 and two sides of 21.8 cm and 25.4 cm.

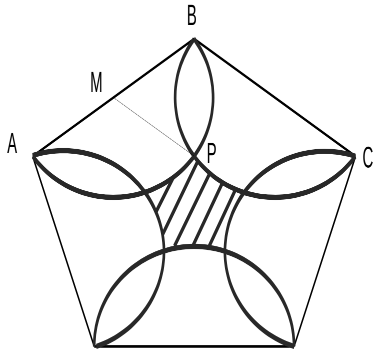
Calculate the length of the third side, correct to 3 significant figures. (4 marks)

$135.5 = 0.5 \times 21.8 \times 25.4 \times \sin\theta$
$\therefore \theta = 150.7°$ (obtuse solution)

$x^2 = 21.8^2 + 25.4^2 - 2 \times 21.8 \times 25.4 \times \cos 150.7°$
$x = 45.67$
$x \approx 45.7$ cm to 3sf

(b) The diagram shows five congruent semicircles standing on the inside of a regular pentagon with sides of length 20 cm.

M is the midpoint of the side AB and P is the point of intersection of two semi-circles.



(i) Show that the size of angle $\angle BMP = 72°$. (2 marks)

BP bisects $\angle ABC$ so that $\angle MBP = 108 \div 2 = 54$

ΔMBP is isosceles, so $\angle BMP = 180 - 2 \times 54 = 72°$

(ii) Determine the area of the central shaded region. (6 marks)

Area of pentagon:

$$5 \times 10 \times \frac{10}{\tan(36)} = 688.19 \text{ cm}^2$$

Area of a semi-circle:

$$\frac{\pi \times 10^2}{2} = 157.08 \text{ cm}^2$$

Area of semi-circle overlap:

$$2 \times \frac{1}{2} \times 10^2 \left(\frac{2\pi}{5} - \sin\left(\frac{2\pi}{5} \right) \right) = 30.56 \text{ cm}^2$$

Required area:

$$688.19 - 5 \times (157.08 - 30.56) = 688.19 - 5 \times 126.52$$

$$= 55.58 \text{ cm}^2$$

Question 14 **(7 marks)**

Consider the function $f(x) = 136 - 96x - 4x^2 + 24x^3 - 4x^4$.

(a) Using calculus techniques, determine the coordinates of all stationary points of the graph of $y = f(x)$ in the interval $0 \le x \le 5$. (4 marks)

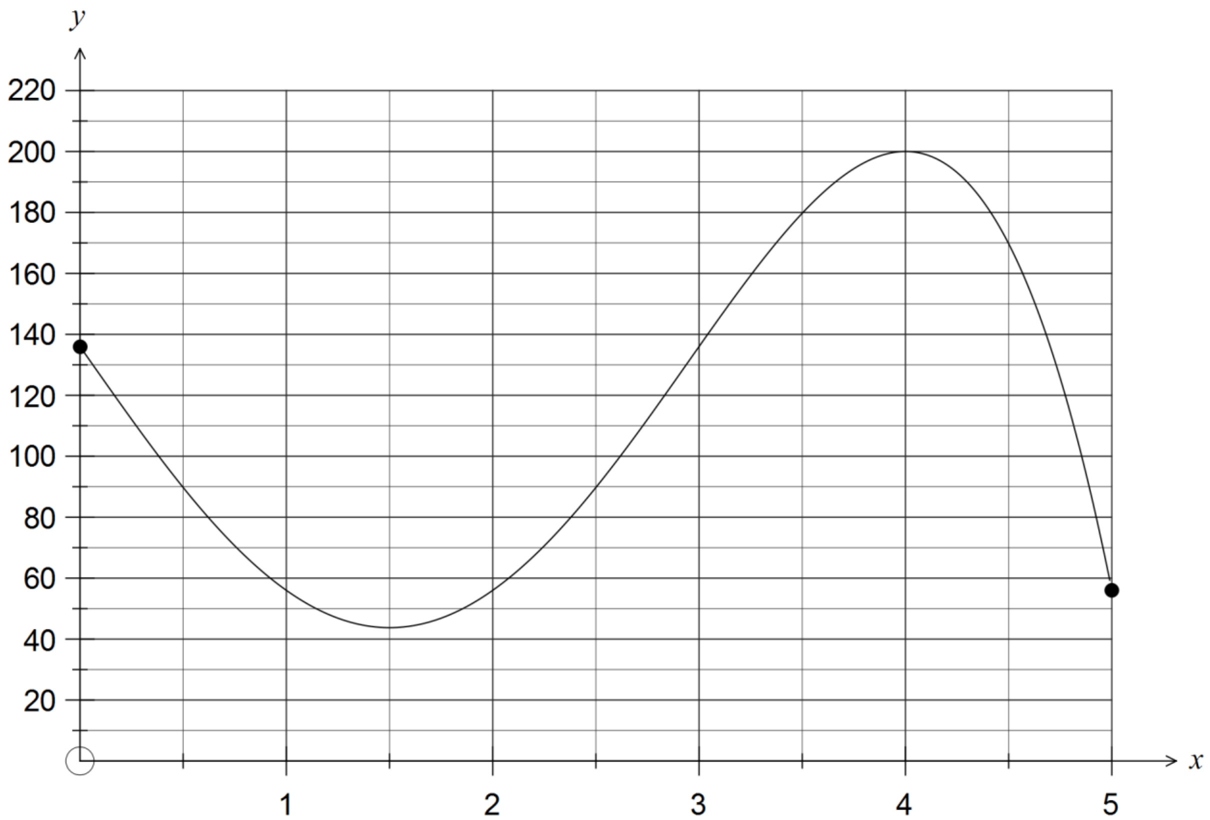
$$\frac{dy}{dx} = -16x^3 + 72x^2 - 8x - 96$$

Solve $0 = -16x^3 + 72x^2 - 8x - 96$

~~$x = -1$~~, $x = 4$, $x = 1.5$

Stationary points at $(4, 200)$ and $(1.5, 43.75)$

(b) Sketch the graph of $y = f(x)$ over the interval $0 \le x \le 5$. (3 marks)



Question 15 **(8 marks)**

A store accepts credit card payments from customers using American Express, Mastercard or VISA cards. Records indicate that 65% of customers use a credit card, and of these customers, 20% use American Express, 35% Mastercard and the rest VISA. Further analysis shows that the male to female ratio for users of each type of card is 5:3 for American Express, 2:3 for Mastercard and 3:2 for VISA.

(a) Calculate the probability that a randomly selected customer from the records will be a female who uses an American Express credit card. (2 marks)

$$0.65 \times 0.2 \times \tfrac{3}{8} = 0.04875$$

(b) Given that a randomly selected customer used a credit card, what is the probability that they are male? (3 marks)

$$\begin{aligned} P(\text{male}) &= 0.2 \times \tfrac{5}{8} + 0.35 \times \tfrac{2}{5} + 0.45 \times \tfrac{3}{5} \\ &= 0.125 + 0.14 + 0.27 \\ &= 0.535 \end{aligned}$$

(c) What is the probability that a randomly selected female customer who used a credit card used a VISA card? (3 marks)

$$\begin{aligned} P(\text{Female} \mid \text{used card}) &= 1 - 0.535 \\ &= 0.465 \end{aligned}$$

$$\begin{aligned} P(\text{Female and VISA} \mid \text{used card}) &= 0.45 \times \tfrac{2}{5} \\ &= 0.18 \end{aligned}$$

$$\begin{aligned} P &= \frac{0.18}{0.465} \\ &= \frac{12}{31} \\ &\approx 0.3871 \end{aligned}$$

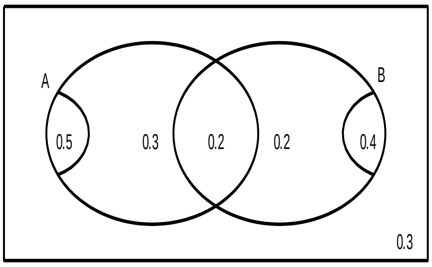
Question 16 **(7 marks)**

Two independent events A and B are such that $\text{P}(\text{A} \cap \text{B}) = 0.2$ and $\text{P}(\bar{\text{B}}) = 0.6$.

(a) Calculate

(i) $\text{P}(\text{A})$ (2 marks)

$$\text{P}(\text{A} \cap \text{B}) = \text{P}(\text{A}) \times \text{P}(\text{B})$$
$$\text{P}(\text{A}) = 0.2 \div 0.4$$
$$= 0.5$$



(ii) $\text{P}(\text{A} \cup \text{B})$ (1 mark)

0.7

(iii) $\text{P}(\bar{\text{B}} \mid (\bar{\text{A}} \cup \bar{\text{B}}))$ (2 marks)

$$\frac{0.6}{0.8} = 0.75$$

(b) A third event, C, is complementary with event A.
What is the maximum possible value of $\text{P}(\text{C} \cup \text{B})$? (2 marks)

$$P(C) = 1 - P(A) = 0.5$$
$$P(B \cap \bar{A}) = 0.2$$

$$0.5 \le P(C \cup (B \cap \bar{A})) \le 0.7$$

Maximum value is 0.7

Question 17 **(7 marks)**

When a capacitor discharges through a resistor, the voltage, V in volts, across the capacitor decays according to the rule $V = 20(0.38)^t$, where t is the time, in seconds, after the discharge began.

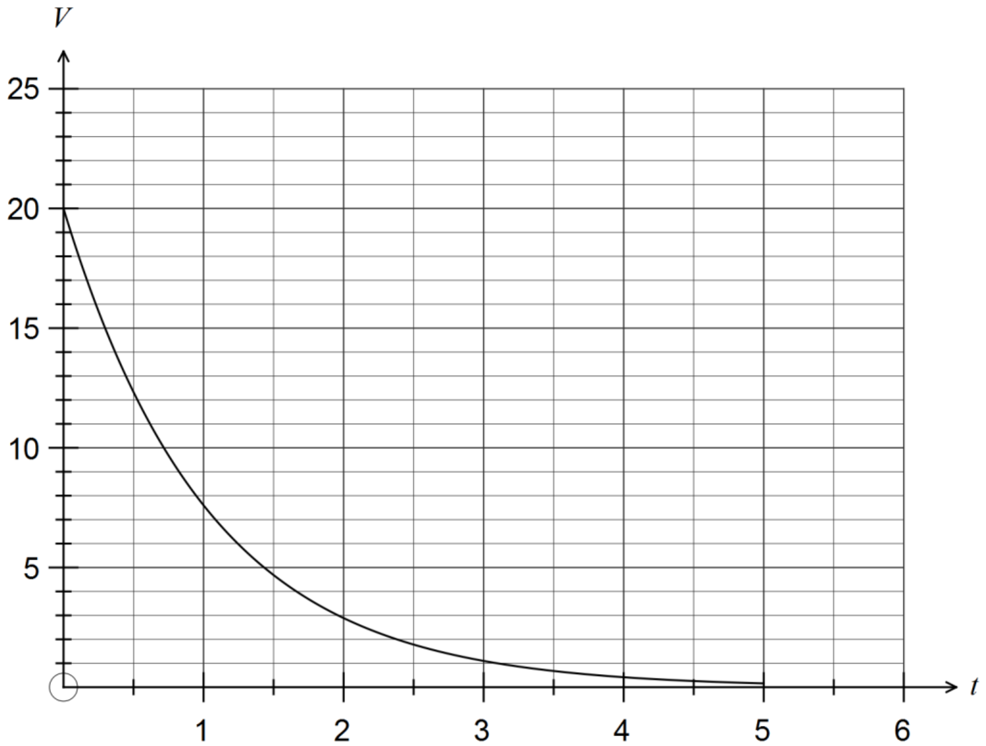
(a) What was the initial voltage across the capacitor? (1 mark)

$V_0 = 20$ volts

(b) What was the voltage across the capacitor after four seconds? (1 mark)

$V = 20(0.38)^4 = 0.417$ volts

(c) Draw the graph of the voltage against time for $0 \le t \le 5$. (3 marks)



(d) How long, to the nearest millisecond, does it take for the voltage across the capacitor to halve? (2 marks)

$20(0.38)^t = 10 \Rightarrow t = 0.716369$

$t = 716$ milliseconds

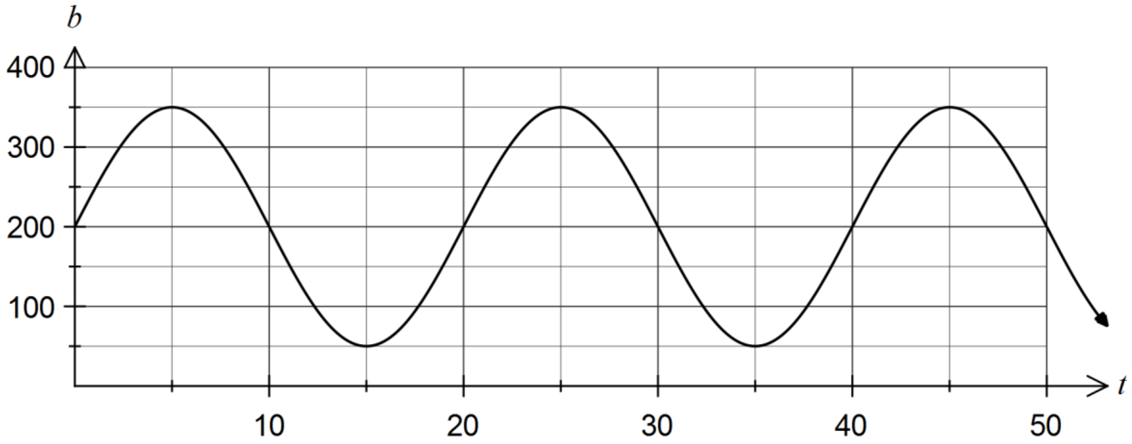
Question 18 **(9 marks)**

The brightness of a small incandescent light globe, b lumens, t milliseconds after measurements began can be modelled by the function $b(t) = r + p\sin(qt)$.

Initially the brightness was 200 lumens, increasing after 5 milliseconds to a maximum of 350 lumens and then dropping to a minimum brightness of 50 lumens after a further 10 milliseconds.

(a) Sketch how the brightness varied over the first 50 milliseconds on the axes below.

(3 marks)



(b) Explain why $p = 150$, $q = \frac{\pi}{10}$ and $r = 200$. (3 marks)

> r is mean of oscillation:
> $r = 200$
>
> q adjusts period to 20 ms:
> $q = \dfrac{2\pi}{20} = \dfrac{\pi}{10}$
>
> p is amplitude of function:
> $350 = 200 + p \Rightarrow p = 150$

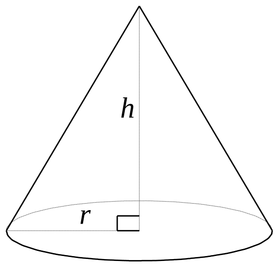
(c) For what percentage of each cycle is the brightness of the globe less than 90 lumens?

(3 marks)

> Solve $90 = 200 + 150\sin(\frac{\pi t}{10})$ to get first two solutions of $t = 12.62$ and $t = 17.38$.
>
> $\dfrac{17.38 - 12.62}{20} \times 100 = 23.8\%$

Question 19 **(6 marks)**

A cone has a radius r and perpendicular height h and is such that the sum of the radius and twice the height is 45 cm.



(a) Show that the volume, V, of the cone is given by $V = \frac{\pi}{3}\left(4h^3 - 180h^2 + 2025h\right)$ cm³.

(3 marks)

$$V = \frac{1}{3}\pi r^2 h$$

But $r + 2h = 45$

$$V = \frac{\pi}{3}(45 - 2h)^2 h$$

$$V = \frac{\pi}{3}\left(4h^3 - 180h^2 + 2025h\right)$$

(b) Using calculus techniques, find the height that will maximise the volume of the cone, and state this maximum volume, rounded to one decimal place. (3 marks)

$$\frac{dV}{dh} = \pi\left(4h^2 - 120h + 675\right)$$

$$\pi\left(4h^2 - 120h + 675\right) = 0$$

$h = 7.5,\ h = 22.5$

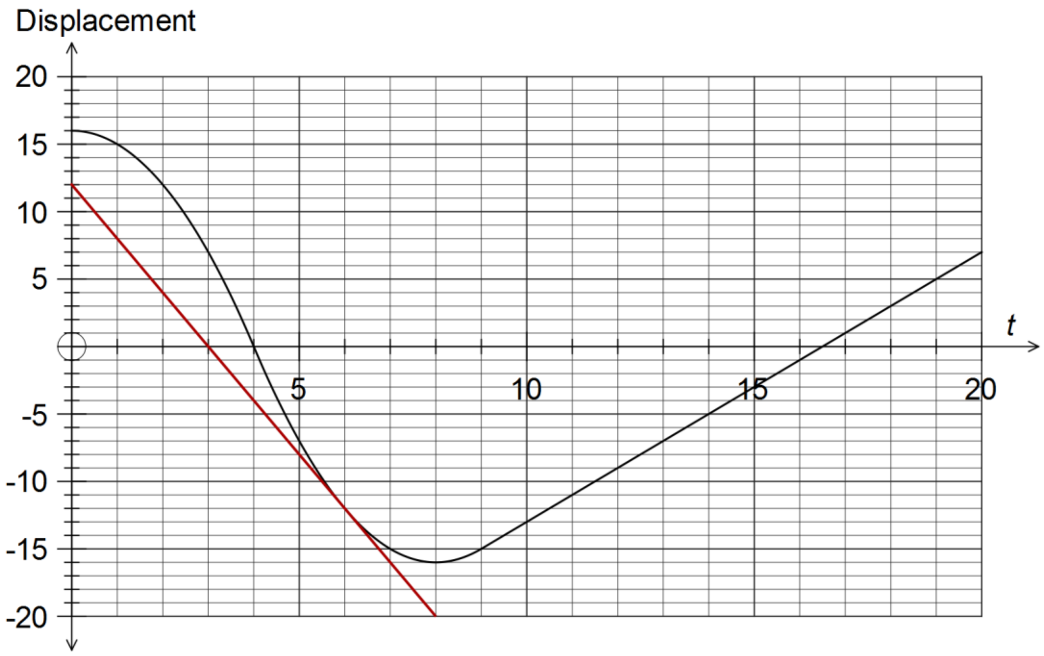
$V(7.5) = 7068.583$

$V(22.5) = 0$

Maximum volume of 7068.6 cm^3 when height is 7.5 cm

Question 20 **(5 marks)**

A small toy train is able to travel backwards and forwards along a straight track built on level ground. The displacement in metres, of the train relative to point A, is shown on the graph below for the interval $0 \le t \le 20$ seconds.



(a) State an interval of time during which the train is moving towards point A. (1 mark)

$0 < t < 4$ or $9 < t < 16.5$

(b) What total distance did the train travel during the 20 second interval? (1 mark)

$16 + 16 + 16 + 7 = 55$ metres

(c) By drawing adding a suitable tangent to the graph above, determine an estimate for the velocity of the train when $t = 6$. (3 marks)

Draw tangent when $t = 6$, as shown.

Gradient: $m \approx -\frac{12}{3} \approx -4$

Estimate is -4 m/s.

End of questions

Additional working space

Question number: _________

Additional working space

Question number: _________

Additional working space

Question number: _________

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